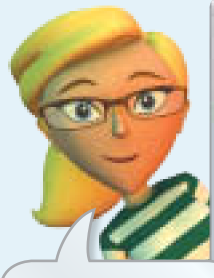
2-1 Patterns and Inductive Reasoning

Common Core State Standards

Prepares for G-CO.C.9 Prove theorems about lines and angles. **Also Prepares for G-CO.C.10, G-CO.C.11**

MP 1, MP 3, MP 4, MP 7

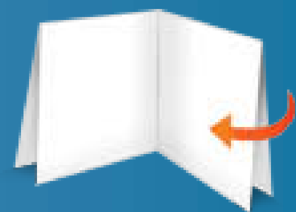
Objective To use inductive reasoning to make conjectures



See if you can find a pattern to help you solve this problem.

SOLVE IT! Getting Ready!

Fold a piece of paper in half. When you unfold it, the paper is divided into two rectangles. Refold the paper, and then fold it in half again. This time when you unfold it, there are four rectangles. How many rectangles would you get if you folded a piece of paper in half eight times? Explain.



MATHEMATICAL PRACTICES In the Solve It, you may have used inductive reasoning. **Inductive reasoning** is reasoning based on patterns you observe.

Essential Understanding You can observe patterns in some number sequences and some sequences of geometric figures to discover relationships.

Plan

How do you *look for a pattern* in a sequence?
Look for a relationship between terms. Test that the relationship is consistent throughout the sequence.

Lesson Vocabulary
- inductive reasoning
- conjecture
- counterexample

Problem 1 Finding and Using a Pattern

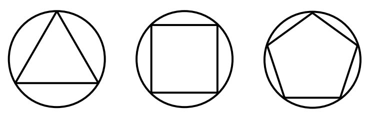
Look for a pattern. What are the next two terms in each sequence?

A 3, 9, 27, 81, . . .

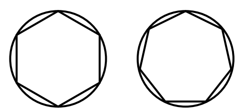
$3 \xrightarrow{\times 3} 9 \xrightarrow{\times 3} 27 \xrightarrow{\times 3} 81$

Each term is three times the previous term. The next two terms are $81 \times 3 = 243$ and $243 \times 3 = 729$.

B



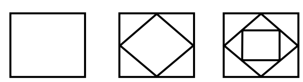
Each circle contains a polygon that has one more side than the preceding polygon. The next two circles contain a six-sided and a seven-sided polygon.



Got It? **1.** What are the next two terms in each sequence?

a. 45, 40, 35, 30, . . .

b.



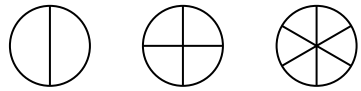
You may want to find the tenth or the one-hundredth term in a sequence. In this case, rather than find every previous term, you can look for a pattern and make a conjecture. A **conjecture** is a conclusion you reach using inductive reasoning.

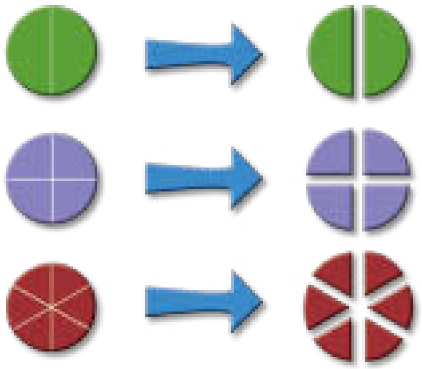
Plan

Do you need to draw a circle with 20 diameters?
No. *Solve a simpler problem* by finding the number of regions formed by 1, 2, and 3 diameters. Then look for a pattern.

Problem 2 Using Inductive Reasoning

Look at the circles. What conjecture can you make about the number of regions 20 diameters form?





1 diameter forms 2 regions.

2 diameters form 4 regions.

3 diameters form 6 regions.

Each circle has twice as many regions as diameters. Twenty diameters form $20 \cdot 2$, or 40 regions.

Got It? 2. What conjecture can you make about the twenty-first term in R, W, B, R, W, B, . . . ?

It is important to gather enough data before you make a conjecture. For example, you do not have enough information about the sequence 1, 3, . . . to make a reasonable conjecture. The next term could be $3 \cdot 3 = 9$ or $3 + 2 = 5$.

Plan

What's the first step?
Start by gathering data. You can organize your data by *making a table*.

Problem 3 Collecting Information to Make a Conjecture

What conjecture can you make about the sum of the first 30 even numbers?

Find the first few sums and look for a pattern.

Number of Terms	Sum
1	$2 = 2 = 1 \cdot 2$
2	$2 + 4 = 6 = 2 \cdot 3$
3	$2 + 4 + 6 = 12 = 3 \cdot 4$
4	$2 + 4 + 6 + 8 = 20 = 4 \cdot 5$

Each sum is the product of the number of terms and the number of terms plus one.

You can conclude that the sum of the first 30 even numbers is $30 \cdot 31$, or 930.

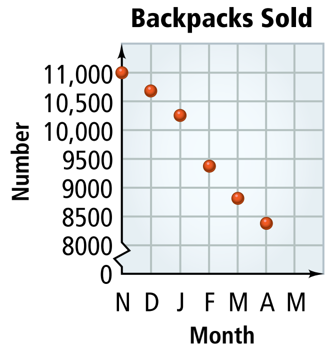
Got It? 3. What conjecture can you make about the sum of the first 30 odd numbers?

Plan

How can you use the given data to make a prediction?
Look for a pattern of points on the graph. Then make a prediction, based on the pattern, about where the next point will be.

Problem 4 Making a Prediction

Sales **Sales of backpacks at a nationwide company decreased over a period of six consecutive months. What conjecture can you make about the number of backpacks the company will sell in May?**



The points seem to fall on a line. The graph shows the number of sales decreasing by about 500 backpacks each month. By inductive reasoning, you can estimate that the company will sell approximately 8000 backpacks in May.

Got It? **4. a.** What conjecture can you make about backpack sales in June?

b. Reasoning Is it reasonable to use this graph to make a conjecture about sales in August? Explain.

Not all conjectures turn out to be true. You should test your conjecture multiple times. You can prove that a conjecture is false by finding *one* counterexample. A **counterexample** is an example that shows that a conjecture is incorrect.

Problem 5 Finding a Counterexample

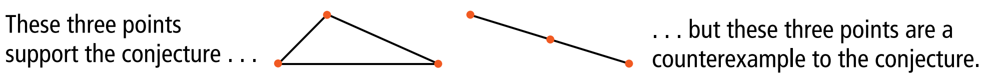
What is a counterexample for each conjecture?

A **If the name of a month starts with the letter J, it is a summer month.**

Counterexample: January starts with J and it is a winter month.

B **You can connect any three points to form a triangle.**

Counterexample: If the three points lie on a line, you cannot form a triangle.



Think

What numbers should you *guess-and-check*?
Try positive numbers, negative numbers, fractions, and special cases like zero.

C **When you multiply a number by 2, the product is greater than the original number.**

The conjecture is true for positive numbers, but it is false for negative numbers and zero.

Counterexample: $-4 \cdot 2 = -8$ and $-8 \ngtr -4$.

Got It? **5.** What is a counterexample for each conjecture?

a. If a flower is red, it is a rose.
b. One and only one plane exists through any three points.
c. When you multiply a number by 3, the product is divisible by 6.

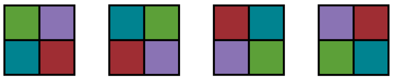
Lesson Check

Do you know HOW?

What are the next two terms in each sequence?

1. 7, 13, 19, 25, . . .

2.



3. What is a counterexample for the following conjecture?

All four-sided figures are squares.

Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. Vocabulary How does the word *counter* help you understand the term *counterexample*?

5. Compare and Contrast Clay thinks the next term in the sequence 2, 4, . . . is 6. Given the same pattern, Ott thinks the next term is 8, and Stacie thinks the next term is 7. What conjecture is each person making? Is there enough information to decide who is correct?

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Find a pattern for each sequence. Use the pattern to show the next two terms. See Problem 1.

6. 5, 10, 20, 40, . . .

7. 1, 4, 9, 16, 25, . . .

8. 1, −1, 2, −2, 3, . . .

9. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

10. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

11. 15, 12, 9, 6, . . .

12. O, T, T, F, F, S, S, E, . . .

13. J, F, M, A, M, . . .

14. 1, 2, 6, 24, 120, . . .

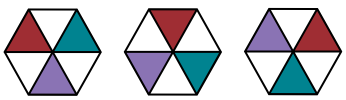
15. Washington, Adams, Jefferson, . . .

16. dollar coin, half dollar, quarter, . . .

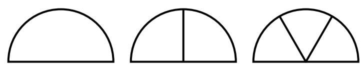
17. AL, AK, AZ, AR, CA, . . .

18. Aquarius, Pisces, Aries, Taurus, . . .

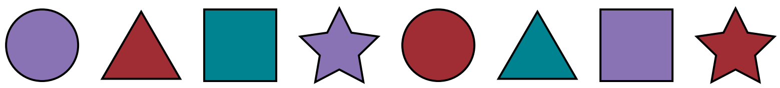
19.



20.



Use the sequence and inductive reasoning to make a conjecture. See Problem 2.



21. What is the color of the fifteenth figure?

22. What is the shape of the twelfth figure?

23. What is the color of the thirtieth figure?

24. What is the shape of the fortieth figure?

Make a conjecture for each scenario. Show your work. See Problem 3.

25. the sum of the first 100 positive odd numbers

26. the sum of the first 100 positive even numbers

27. the sum of two odd numbers

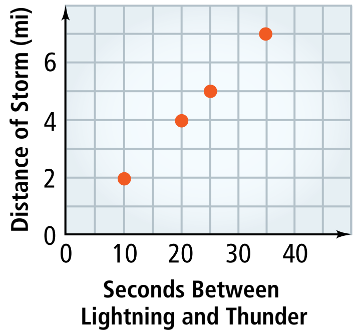
28. the sum of an even and odd number

29. the product of two even numbers

30. the product of two odd numbers

STEM **Weather** **Use inductive reasoning to make a prediction about the weather.** See Problem 4.

31. Lightning travels much faster than thunder, so you see lightning before you hear thunder. If you count 5 s between the lightning and thunder, how far away is the storm?



32. The speed at which a cricket chirps is affected by the temperature. If you hear 20 cricket chirps in 14 s, what is the temperature?

Number of Chirps per 14 Seconds	Temperature (°F)
5	45
10	55
15	65

Find one counterexample to show that each conjecture is false. See Problem 5.

33. $\angle 1$ and $\angle 2$ are supplementary, so one of the angles is acute.

34. $\triangle ABC$ is a right triangle, so $\angle A$ measures 90.

35. The sum of two numbers is greater than either number.

36. The product of two positive numbers is greater than either number.

37. The difference of two integers is less than either integer.

B Apply

Find a pattern for each sequence. Use inductive reasoning to show the next two terms.

38. 1, 3, 7, 13, 21, . . .

39. 1, 2, 5, 6, 9, . . .

40. 0.1, 0.01, 0.001, . . .

41. 2, 6, 7, 21, 22, 66, 67, . . .

42. 1, 3, 7, 15, 31, . . .

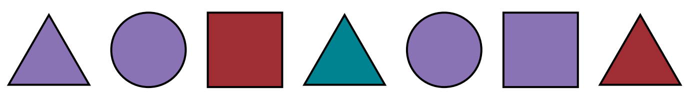
43. $0, \frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \ldots$

Predict the next term in each sequence. Use your calculator to verify your answer.

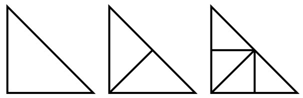
44.
$$\begin{aligned}
12345679 \times 9 &= 111111111\\
12345679 \times 18 &= 222222222\\
12345679 \times 27 &= 333333333\\
12345679 \times 36 &= 444444444\\
12345679 \times 45 &= \blacksquare
\end{aligned}$$

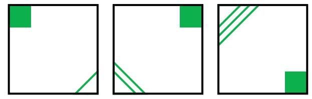
45.
$$\begin{aligned}
1 \times 1 &= 1\\
11 \times 11 &= 121\\
111 \times 111 &= 12321\\
1111 \times 1111 &= 1234321\\
11111 \times 11111 &= \blacksquare
\end{aligned}$$

46. **Patterns** Draw the next figure in the sequence. Make sure you think about color and shape.

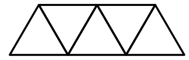


Draw the next figure in each sequence.

47. 

48. 

49. Reasoning Find the perimeter when 100 triangles are put together in the pattern shown. Assume that all triangle sides are 1 cm long.



50. Think About a Plan Below are 15 points. Most of the points fit a pattern. Which does not? Explain.

$A(6, -2)$ $B(6, 5)$ $C(8, 0)$ $D(8, 7)$ $E(10, 2)$ $F(10, 6)$ $G(11, 4)$ $H(12, 3)$
$I(4, 0)$ $J(7, 6)$ $K(5, 6)$ $L(4, 7)$ $M(2, 2)$ $N(1, 4)$ $O(2, 6)$

- How can you draw a diagram to help you find a pattern?
- What pattern do the majority of the points fit?

51. Language Look for a pattern in the Chinese number system.

a. What is the Chinese name for the numbers 43, 67, and 84?

b. Reasoning Do you think that the Chinese number system is base 10? Explain.

Chinese Number System

Number	Chinese Word	Number	Chinese Word
1	yī	9	jǐu
2	èr	10	shí
3	sān	11	shí-yī
4	sì	12	shí-èr
5	wǔ	⋮	⋮
6	lìu	20	èr-shí
7	qī	21	èr-shí-yī
8	bā	⋮	⋮
		30	sān-shí

52. Open-Ended Write two different number sequences that begin with the same two numbers.

53. Error Analysis For each of the past four years, Paulo has grown 2 in. every year. He is now 16 years old and is 5 ft 10 in. tall. He figures that when he is 22 years old he will be 6 ft 10 in. tall. What would you tell Paulo about his conjecture?

STEM **54. Bird Migration** During bird migration, volunteers get up early on Bird Day to record the number of bird species they observe in their community during a 24-h period. Results are posted online to help scientists and students track the migration.

a. Make a graph of the data.

b. Use the graph and inductive reasoning to make a conjecture about the number of bird species the volunteers in this community will observe in 2015.

Bird Count

Year	Number of Species
2004	70
2005	83
2006	80
2007	85
2008	90

55. Writing Describe a real-life situation in which you recently used inductive reasoning.

Challenge

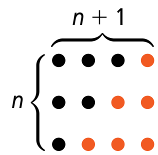
56. History When he was in the third grade, German mathematician Karl Gauss (1777–1855) took ten seconds to sum the integers from 1 to 100. Now it's your turn. Find a fast way to sum the integers from 1 to 100. Find a fast way to sum the integers from 1 to n. (*Hint:* Use patterns.)

57. Chess The small squares on a chessboard can be combined to form larger squares. For example, there are sixty-four 1×1 squares and one 8×8 square. Use inductive reasoning to determine how many 2×2 squares, 3×3 squares, and so on, are on a chessboard. What is the total number of squares on a chessboard?

58. a. Algebra Write the first six terms of the sequence that starts with 1, and for which the difference between consecutive terms is first 2, and then 3, 4, 5, and 6.

b. Evaluate $\frac{n^2 + n}{2}$ for $n = 1, 2, 3, 4, 5$, and 6. Compare the sequence you get with your answer for part (a).

c. Examine the diagram at the right and explain how it illustrates a value of $\frac{n^2 + n}{2}$.



d. Draw a similar diagram to represent $\frac{n^2 + n}{2}$ for $n = 5$.

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 3

Look back at the information about 2-by-2 calendar squares on page 81. The March calendar from page 81 is shown again below.

MARCH						
SUN	MON	TUE	WED	THU	FRI	SAT
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

a. Use the 2-by-2 calendar square outlined in red on the March calendar. Find a relationship between the pairs of numbers on the diagonals of the calendar square.

b. Repeat part (a) for several other 2-by-2 calendar squares on the March calendar. Make a conjecture based on your results.

c. Test your conjecture using other calendar months. Can you find a counterexample to your conjecture?